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Spontaneous acoustic emission from strong ionizing shocks

By M. MOND AND I. M. RUTKEVICH

Department of Mechanical Engineering, Ben-Gurion University of the Negev, PO Box 653, Beer Sheva 84105, Israel

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Dyakov (1954) and Kontorovich (1957) formulated the conditions for corrugation instability of shock waves as well as for spontaneous emission of sound and entropy-vortex waves from them. For the first time since then, it is shown here that physical circumstances do exist under which shocks in gases spontaneously emit sound waves. Such circumstances are provided by strong ionizing shocks. In order to see that, the coefficient of reflection of an acoustic wave from a shock is derived as a function of the wave's frequency and the ionization degree. Spontaneous emission of sound occurs when the reflection coefficient becomes infinitely large. It is shown that the relevant frequency range for the occurrence of spontaneous emission is that for which the electrons are not in local thermodynamic equilibrium with the heavy particles. The special properties of acoustic perturbations behind the ionizing shock are considered for this frequency range and the sound velocity in a partially ionized gas is derived. In addition, the condition for spontaneous emission of sound is modified in order to take into account the difference between the electrons and heavy-particle perturbed temperatures. It is shown, by numerical calculations, that the criterion for spontaneous emission is satisfied behind ionizing shocks in argon. In particular, for an initial pressure of 5 Torr, the threshold for the occurrence of the spontaneous emission is found to be $M_1 = 15$. This critical value of the shock Mach number, as well as other calculated physical features, agree very well with those obtained experimentally by Glass & Liu (1978) who observed the occurrence of instability behind shocks in argon.

1. Introduction

The problem of stability of plane shock waves propagating in an unbounded medium has attracted the attention of researchers over the last forty years. Dyakov (1954) first formulated the problem of corrugation instability of a shock. This instability is manifested in the growth of sinusoidal deformations of the plane front of a shock. The criterion of corrugation instability was obtained by Dyakov in the form of inequalities for a certain parameter h, which contains the derivative of the specific volume with respect to the pressure along the shock adiabatic. He also found the range of values of the parameter h in which small acoustic and entropy-vortex perturbations in the form of sinusoidal two-dimensional waves can be emitted by the shock. This phenomenon was termed spontaneous emission.

The critical value h_c of the parameter h corresponding to the occurrence of spontaneous emission was determined by Kontorovich (1957, 1959). Dyakov and Kontorovich treated this effect as a resonant reflection when the reflection coefficient $\mathscr{R}$ of the acoustic wave from the shock takes an infinitely large value for a certain direction of the wave vector of the incident wave.

Although in the case of spontaneous emission the eigenoscillations do not grow in time, their occurrence undoubtedly indicates that a real flow behind the spontaneously emitting shock is unstable. The reason for this instability is in the fact that any real system has a finite size, so that the initial energy of the shock is finite. Therefore, a permanent loss of the shock's energy due to the continuous radiation of outgoing waves will result in reorganization of the initial flow. In principle, even the condition $|\mathscr{R}| > 1$ may be considered as an indication of instability, since in this case the shock is working as an amplifier of incident waves. As was shown by Fowles (1981), if $|\mathscr{R}| > 1$ for a certain angle of incidence, then some other angle of incidence exists, for which $\mathscr{R} = \infty$.

Neither corrugation instability nor spontaneous emission can occur in a perfect polytropic gas with a constant adiabatic index γ. As regards to the shock instabilities in real gases, since the formulation of the classical Dyakov–Kontorovich criteria, no appropriate physical conditions have been found under which the criteria are satisfied. It would be expected that strong dissociating or ionizing shocks might be spontaneously emitting according to the classical criterion due to the non-monotonic forms of their shock adiabatics. Such shock adiabatics were obtained in experiments by Griffiths, Sandeman & Hornung (1976) with ionizing shocks in argon and with dissociating shocks in carbon dioxide. These authors observed various forms of shock-induced instabilities. However, even for non-monotonic shock adiabatics sampling numerical calculations performed on the basis of the classical criterion of spontaneous emission did not confirm their existence, although in many cases the values h and h_c were close (Griffiths *et al.* 1976; Kuznetsov 1989).

The development of instabilities behind strong shocks in inert gases has been observed in shock-tube experiments with argon (Glass & Liu 1978) and krypton (Glass, Liu & Tang 1977). An interferometer having a laser light source was used in the experiment, and the simultaneous interferograms for two different wavelengths made it possible to determine both gas density ρ and electron concentration n_e in the flow. Spatial oscillations in the distributions of ρ and n_e were observed in these experiments in the region behind the relaxation zone when the shock Mach number was sufficiently large: $M_1 \geqslant 15$.

The goal of present paper is a self-consistent consideration of the problem of stability of ionizing shocks in pure monatomic gases. As will be shown, such a consideration should take into account the ionization kinetics influencing the properties of acoustic waves in a partially ionized plasma produced by a strong shock. The investigation of the instability is based on the analysis of the reflection coefficient $\mathscr{R}$ with an emphasis on the condition of resonant reflection. However, the quantity $\mathscr{R}$ derived in the current work actually differs from the expression for $\mathscr{R}$ given in previous works. Thus, the dependence of the ionization degree upon the electron temperature and the gas density influences not only the form of the stationary shock adiabatic, but also the form of the dynamic relations between the perturbations of the various electron gas parameters.

A modification of the reflection coefficient leads to a new definition of the parameter h that determines the relationship between the density and pressure perturbations behind the ionizing shock. The critical value $h = h_c$ also is changed. The reconsidered criterion for spontaneous emission predicts the existence of a critical value of the shock Mach number M_1, beyond which spontaneous emission should occur. This result is in qualitative agreement with existing experiments.

The paper is organized as follows. In §2 the formulation of the problem of the interaction of a shock wave with small perturbations is given. Different types of elementary two-dimensional waves are described, and the decomposition of a general problem into four independent problems is discussed. The reflection coefficient $\mathscr{R}$ is defined for two-dimensional acoustic waves varying in time as $\exp(-\mathrm{i}\omega t)$. An explicit dependence of $\mathscr{R}$ on the frequency ω is presented and employed for obtaining the classical criterion of spontaneous emission. The advantages of introducing the frequency-dependent representation $\mathscr{R}(\omega)$ in the complex ω-plane are discussed. In particular, it is shown that the classical criteria for corrugation instability can be obtained from the condition of resonant reflection $\mathscr{R}(\omega) = \infty$ for acoustic waves of a special type. These waves grow in time and decay exponentially in space in the direction of their propagation.

Section 3 contains the description of ionizing shock adiabatics, where the concept of a unified shock that consists of the gasdynamic shock and the relaxation zone is employed. The conservation laws for an ionizing shock are considered, and a system of relations for determining a unified shock adiabatic is given. This system includes the Saha equation for the degree of ionization.

An implicit parametric form of the shock adiabatic equation is presented in an analytical form. It is shown that the dependence of the density upon the pressure representing the ionizing shock adiabatic always possesses a maximum corresponding to the maximal degree of compression produced by the shock. The general considerations are supplemented by results of numerical calculation of an ionizing shock adiabatic in argon.

In addition, a mutual arrangement of the ionizing shock adiabatic, the gasdynamic shock adiabatic, and the Rayleigh line in the pressure–density plane is considered in this section. The significance of the Rayleigh line for a true choice of the physically reasonable solution describing the equilibrium state behind the relaxation zone is discussed.

Section 4 gives the analysis of small perturbations in a partially ionized gas behind the relaxation zone. The influence of ionization by electron–atom collisions on the sound velocity and the reflection coefficient is considered. It is shown that acoustic properties of a partially ionized gas depend not only on the frequency ω, but also on the ionization degree α as well as on partial derivatives of the latter with respect to the gas density and temperature in the equilibrium state. In addition, the revised criterion for spontaneous emission is obtained for ionizing shocks.

In §5 the criterion for spontaneous emission is examined numerically for strong shocks in argon. The existence of threshold for the occurrence of spontaneous emission with respect to the shock Mach number M_1 is found and the results of the corresponding numerical calculations are presented. The calculated critical value of the Mach number is in good agreement with the experiment of Glass & Liu (1978).

Section 6 contains a discussion of the main results including a comparison of the proposed theory with experiments, comments on the validity of the model of ionizing shocks, and justification of the assumptions that are made in the description of the acoustic waves in a partially ionized gas.

2. Reflection coefficient for an acoustic wave

2.1. *Interaction of small perturbations with a shock*

In analysing the interactions of small perturbations with a shock wave, it is convenient to describe these interactions in the frame of reference $\mathscr{K}$ moving with the normal velocity of the unperturbed shock. The system $\mathscr{K}$ always can be chosen so that the gas

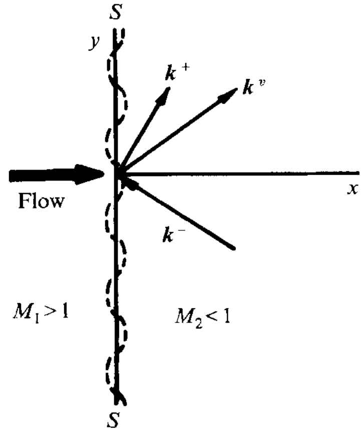

FIGURE 1. Sketch of the interaction of a shock with small perturbations. S is the shock surface: ——, unperturbed shock; ----, perturbed shock; $\boldsymbol{k}^-, \boldsymbol{k}^+$ and $\boldsymbol{k}^v$ are the wave vectors of the incident acoustic wave, reflected acoustic wave and entropy-vortex wave, respectively.

velocity ahead of the shock is directed normally to the latter as is shown in figure 1. Considering small perturbations in the form

$$\exp[\mathrm{i}(k_x x + \boldsymbol{k}_\perp \cdot \boldsymbol{r} - \omega t)], \tag{2.1}$$

where $\boldsymbol{k}_\perp$ is the projection of the wavevector $\boldsymbol{k}$ on the unperturbed shock plane S, we choose the y-axis of the coordinate system $\mathscr{K}$ to be directed along $\boldsymbol{k}_\perp$, so that in (2.1), $\boldsymbol{k}_\perp \cdot \boldsymbol{r} = k_y y$.

Following Landau & Lifshitz (1987), we assume the conditions

$$M_1 = V_1/c_1 > 1, \quad M_2 = V_2/c_2 < 1. \tag{2.2}$$

Here and below the subscripts 1 and 2 refer to the states ahead of the shock and behind it, respectively. In (2.2), c is the sound velocity, V is the gas velocity, and M is the Mach number. Conditions (2.2) are always satisfied for shocks in a perfect gas. For real gases these conditions should be imposed in accordance with the requirement of the evolutionarity of a shock wave.

In region 1 we consider a uniform stationary flow satisfying the system of Euler equations for an ideal, non-viscous gas. In region 2 these equations should be supplemented with the relations describing the electron gas. Generally, for a shock that may induce chemical reactions of various types, the flow in region 2 should be described by the equations of high-temperature gasdynamics (Anderson 1989). Also, we assume that in region 2 the gas is in a chemical equilibrium and the unperturbed flow in this region is stationary and uniform. This allows us to consider the solutions of the type (2.1) with $\boldsymbol{k} = \text{const}$ in both regions 1 and 2. The concept of a shock wave that is employed in the present consideration includes the usual gasdynamic shock and the relaxation zone behind it, in which the electron density increases to the value corresponding to the condition of ionization equilibrium. The relaxation zone is terminated by the equilibrium region 2 as is shown in figure 2. In analysing the interactions of small perturbations with an ionizing shock we may replace the shock and the relaxation zone by a single surface of discontinuity. That is admissible if the wavelength is larger than the characteristic thickness $\varDelta$ of the relaxation zone:

$$\varDelta < 1/|k_x|. \tag{2.3}$$

When the influence of viscosity and heat conduction on the perturbations are neglected in both regions 1 and 2, the perturbations are described by equations of

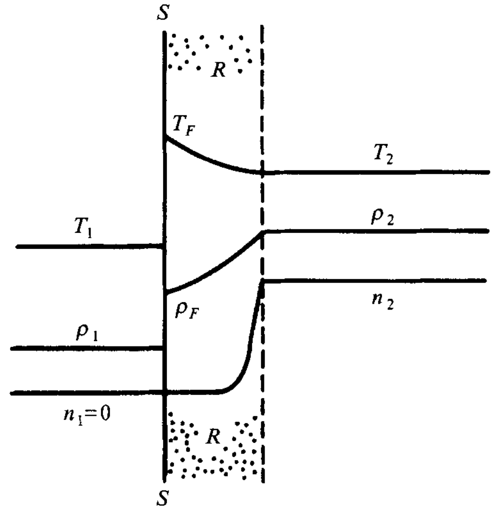

FIGURE 2. Behaviour of the gas temperature T, gas density ρ and electron concentration n in the relaxation zone R. Subscripts 1, F and 2 refer to the initial state, the frozen and equilibrium states behind the shock, respectively.

hyperbolic type. Then in the high-frequency limit ($\omega \rightarrow \infty$) the dispersion relation for acoustic waves takes the form

$$\left(\frac{\omega}{k} - \boldsymbol{n} \cdot \boldsymbol{V}\right)^2 = c^2, \quad \boldsymbol{n} = \frac{\boldsymbol{k}}{k} = \frac{\boldsymbol{k}}{(\boldsymbol{k} \cdot \boldsymbol{k})^{1/2}}. \tag{2.4}$$

The high-frequency sound velocity appearing in (2.4) is known in the dynamics of real gases as frozen velocity (Anderson 1989). This term emphasizes the fact that the fast acoustic oscillations cannot perturb the gas mixture species when the characteristic time of the transition to chemical equilibrium becomes very long compared to the period of oscillations.

When conditions (2.2) are violated, the problem of small perturbations – shock wave interactions regarded as an initial-boundary problem becomes ill-posed. For given initial perturbations, the solution that should describe their evolution for $t > 0$ either does not exist, or cannot be uniquely determined. Such shock waves, called non-evolutionary (Landau & Lifshitz 1987), are not considered here.

If small perturbations are absent at $t = 0$ in the supersonic region 1, they never can occur in this region at $t > 0$. On the other hand, in the subsonic region 2 the perturbations may appear (even if they were absent initially) in the form of acoustic and entropy-vortex waves coming from the supersonic region 1 after their transmission through the shock.

A general problem of the interaction of a shock with small perturbations can be decomposed into four separate linear problems, so that each of them may be considered independently of the others. These problems refer to the following situations:

(*a*) An acoustic wave excited in the subsonic region 2 propagates towards the shock and interacts with it, resulting in the appearance of a reflected downstream acoustic wave and a generated entropy-vortex wave. Both these waves propagate in region 2 away from the shock as is shown in figure 1.

(*b*) A downstream acoustic wave excited in the supersonic region 1 interacts with the shock and generates a downstream acoustic wave and an entropy-vortex wave propagating in region 2 away from the shock.

(*c*) The situation is analogous to (*b*) with the distinction that the incident acoustic wave propagates upstream in region 1. Since the group velocity of this wave is directed downstream, the wave will interact with the shock. (Such a situation may occur in a shock-tube experiment when the shock and the sound wave propagate in an immovable gas before the shock in the same direction, while the shock overtakes the wave.)

(*d*) An entropy-vortex wave is excited in region 1. This wave interacts with the shock and generates a downstream acoustic wave and a new entropy-vortex wave in region 2.

In this section we shall neglect the influence of the relaxation processes behind the shock on the behaviour of small perturbations. Then, relation (2.4) may be used not only in the high-frequency limit but for arbitrary frequencies. The corresponding dispersion relation for entropy-vortex waves is

$$\omega = \boldsymbol{k}\cdot\boldsymbol{V}. \tag{2.5}$$

We denote the upstream and downstream acoustic waves, and the entropy-vortex waves as $\mathscr{A}^-$, $\mathscr{A}^+$ and $\mathscr{A}^{(v)}$, respectively. An acoustic wave is characterized by a single variable, say the pressure perturbation, δp. For such a wave, the entropy and vorticity perturbations are zero, while the velocity perturbation is expressed in terms of δp as follows:

$$\delta \boldsymbol{V}^{\mp} = \frac{\delta p^{\mp}}{\rho c}\boldsymbol{n}^{\mp}, \quad \boldsymbol{n}^{\mp} = \frac{\boldsymbol{k}^{\mp}}{(\boldsymbol{k}^{\mp}\cdot\boldsymbol{k}^{\mp})^{1/2}}. \tag{2.6}$$

The upper and lower superscripts in (2.6) refer to the upstream and downstream waves, respectively. In the relations (2.4)–(2.6) $\boldsymbol{k}$ and ω are generally complex-valued quantities. An entropy-vortex wave is determined by three independent variables, the perturbation of the entropy δS, and the perturbation of the vorticity $\delta\boldsymbol{\Omega}$. The latter has two independent components due to the relation $\boldsymbol{n}\cdot\delta\boldsymbol{\Omega} = 0$. For an entropy-vortex wave, the pressure perturbation is zero, while the density and velocity perturbations are expressed in terms of δS and $\delta\boldsymbol{\Omega}$ as follows:

$$\delta\rho^{(s)} = \left(\frac{\partial\rho}{\partial S}\right)_p \delta S, \quad \delta\boldsymbol{V}^{(v)} = \frac{\delta\boldsymbol{\Omega}\times\boldsymbol{k}}{\boldsymbol{k}\cdot\boldsymbol{k}}. \tag{2.7}$$

We concentrate below on problem (*a*), which is formulated as follows. Find the solution of the linearized gasdynamic Euler equations satisfying the linearized conservation laws at the shock surface and the conditions

$$\left.\begin{aligned} &\delta p_1 = 0, \quad \delta S_1 = 0, \quad \delta\Omega_1 = 0 \quad (x<0),\\ &\delta p_2 = \delta p^+ + \delta p^- \neq 0, \quad \delta S_2 \neq 0, \quad \delta\Omega_2 \neq 0 \quad (x>0).\end{aligned}\right\} \tag{2.8}$$

The various perturbations in the subsonic flow may be written in the form

$$\left.\begin{aligned} &\delta p_2^{\pm} = C^{\pm}\exp\left[\mathrm{i}(k_x^{\pm}x + k_y y - \omega t)\right] \quad (C^{\pm} = \text{const}),\\ &\{\delta S, \delta\Omega\} = \{S_0, \Omega_0\}\exp\left[\mathrm{i}(k_x^{(v)}x + k_y y - \omega t)\right] \quad (S_0 = \text{const}, \quad \Omega_0 = \text{const}).\end{aligned}\right\} \tag{2.9}$$

For given values of ω and k_y, the longitudinal components of the wave vectors of these three waves are determined by the expressions

$$\left.\begin{aligned} &k_x^{\pm} = \frac{\omega[-M_2 \pm (1-q^{-2})^{1/2}]}{c_2(1-M_2^2)}, \quad q^{-2} = \frac{\omega_c^2}{\omega^2}, \quad \omega_c^2 = c_2^2 k_y^2(1-M_2^2),\\ &k_x^{(v)} = \frac{\omega}{M_2 c_2}.\end{aligned}\right\} \tag{2.10}$$

Here ω_c is a critical (cutoff) frequency for two-dimensional acoustic waves. For a given value of k_y, the acoustic waves with real ω can propagate in a subsonic flow only if $\omega^2 > \omega_c^2$.

For a given amplitude of the incident wave C^-, the amplitude of the reflected acoustic wave is determined by

$$C^+_\backslash = \mathscr{R}(k_y, \omega)\, C^-, \tag{2.11}$$

where $\mathscr{R} = (\delta p_2^+/\delta p_2^-)_{x=0}$ is the reflection coefficient. In addition to its dependence on k_y and ω, the reflection coefficient $\mathscr{R}$ also depends on the specific properties of the shock adiabatic.

2.2. *Frequency-dependent representation of the reflection coefficient*

The problem of determining the reflection coefficient for shocks in a gas with arbitrary form of shock adiabatic was considered by Kontorovich (1959). The first attempt to derive this coefficient for two-dimensional waves was made by Brillouin (1953). As is mentioned in Kontorovich's work, Brillouin did not take into account the sinusoidal perturbations of the shock surface. The contribution of the latter to the value of $\mathscr{R}$ may be of the same order as the other perturbations behind the shock. Kontorovich (1959) derived a system of relations from which the expression for $\mathscr{R}$ can be obtained in terms of the angles between the following pair of vectors: $\{\boldsymbol{k}^-, \boldsymbol{e}_x\}$, $\{\boldsymbol{k}^+, \boldsymbol{e}_x\}$ and $\{\boldsymbol{k}^-, \boldsymbol{V}_g^-\}$. Here $\boldsymbol{e}_x$ is the unit vector in the x-direction and $\boldsymbol{V}_g^-$ is the group velocity of the incident acoustic wave,

$$\boldsymbol{V}_g^- = \boldsymbol{V}_2 + c_2 \boldsymbol{n}_2^-. \tag{2.12}$$

Although an explicit formula for $\mathscr{R}$ was given by Kontorovich only for the case of normal wave incidence ($k_y = 0$), he took into account an oblique incidence when he considered the region of parameters in which $\mathscr{R}$ becomes infinitely large for a given direction $\boldsymbol{n}_2^-$.

The procedure for obtaining the expression for $\mathscr{R}$ is based on the linearized equations of gasdynamics and the linearized Rankine–Hugoniot relations. These relations can be found in Dyakov (1954) and Kontorovich (1959), as well as in Landau & Lifshitz (1987). Modifying the approach developed in these works, we employ the frequency-dependent representation of the reflection coefficient. Such a representation has the form

$$\mathscr{R} = -\frac{f(q) - (q^2-1)^{1/2}}{f(q) + (q^2-1)^{1/2}}, \quad f = \frac{1-h}{2M_2} q - \frac{(1+h)\,\eta M_2}{2q(1-M_2^2)}, \tag{2.13}$$

where

$$\left.\begin{aligned} q = \frac{\omega}{\omega_c} = \frac{\omega}{k_y c_2 (1-M^2)^{1/2}}, \quad h = -V_2^2\left(\frac{\partial \rho_2}{\partial p_2}\right)_{\rho_1, p_1} = -\frac{j^2}{\rho_1 p_1 \eta^2}\left(\frac{\partial \eta}{\partial P}\right)_{\rho_1, p_1}, \\ \eta = \rho_2/\rho_1, \quad P = p_2/p_1, \quad j = \rho_1 V_1 = \rho_2 V_2. \end{aligned}\right\} \tag{2.14}$$

Here η is the density ratio, and P is the pressure ratio for the quantities taken at different sides of the shock, j is the mass flow rate across the shock. The variables η and P are connected through

$$\eta = \eta(P), \tag{2.15}$$

called the shock adiabatic (or the Hugoniot curve). The quantity η also depends on some dimensionless parameters characterizing the initial state of the gas ahead of the shock, as well as the parameters characterizing the chemical reactions when they are induced by the shock.

As is seen from (2.13), the reflection coefficient $\mathscr{R}(\omega, k_y)$ actually depends only on the ratio ω/k_y. Instead of this ratio one may use any function depending on ω/k_y, for instance the incidence angle of the wave $\mathscr{A}^-$. Thus, in the work of Fowles (1981), who considered the reflection of weak discontinuities from a shock, the reflection coefficient was calculated in terms of the angle of incidence.

An explicit frequency-dependent representation of $\mathscr{R}$ (2.13) is convenient in solving various boundary-value problems, which include calculating the reflection of acoustic waves from shocks as a necessary step in obtaining the solution. For example, one such problem is the investigation of the spectrum of eigenperturbations in the subsonic region between the shock and some other reflecting surface (Rutkevich & Mond 1992). The advantages of using the dependence $\mathscr{R}(\omega, k_y)$ become evident when q is a complex-valued quantity and, therefore, a simple geometrical concept of the angle of incidence fails.

In the next subsection we show that the representation (2.13) is useful in deriving the conditions for spontaneous emission and corrugation instability.

2.3. *Singularity of the reflection coefficient as an indication of spontaneous emission and corrugation instability*

In the work of Dyakov (1954) spontaneous emission of acoustic waves from a shock was identified by the existence of solutions given by (2.9), for which $C^- = 0$ and $C^+ \neq 0$ at certain real values of $\omega, \boldsymbol{k}^+$ and $\boldsymbol{k}^{(v)}$. Such solutions that represent outgoing waves satisfy all the boundary conditions. In accordance with (2.11), this can exist only if $\mathscr{R} = \infty$. Therefore, the phenomenon of spontaneous emission may be treated as a special case of reflection where $\mathscr{R} = \infty$. The criterion of spontaneous emission obtained by Kontorovich (1959) has the form

$$h_c \leqslant h \leqslant 1 + 2M_2, \quad h_c = \frac{1-(1+\eta)M_2^2}{1+(\eta-1)M_2^2}. \tag{2.16}$$

Here, h is the parameter defined in (2.14), and h_c is the critical value of h corresponding to the threshold of spontaneous emission occurrence. Condition (2.16) can be derived easily from the requirement $\mathscr{R} = \infty$ with the aid of (2.13). Assuming that the denominator on the right-hand side of (2.13) equals zero, we get the relation

$$h = H(q, M_2) \equiv \frac{(1-M_2^2)[q^2 + 2M_2 q(q^2-1)^{1/2}] - \eta M_2^2}{(1-M_2^2)q^2 + \eta M_2^2}. \tag{2.17}$$

For outgoing waves with real values of ω and $\boldsymbol{k}^+$, the parameter q satisfies

$$1 \leqslant q \leqslant \infty. \tag{2.18}$$

Consider the right-hand side of (2.17) as a function of q. This function is monotonically increasing, while $H = h_c$ at $q = 1$, and $H \rightarrow 1 + 2M_2$ as $q \rightarrow \infty$. The function $H(q)$ is shown schematically in figure 3(a).

Considering now relation (2.17) as an equation for determining q, we see that for a given value of h, a unique solution $q = \hat{q}(h)$ with $1 \leqslant \hat{q}(h) \leqslant \infty$ exists if and only if h satisfies the condition $h_c \leqslant h \leqslant 1 + 2M_2$. This is exactly condition (2.16). For each value of h belonging to the interval (2.16), the value $\hat{q}(h)$ determines a unit vector $\boldsymbol{n}^+$ and a group velocity V_g^+ of the emitted acoustic wave:

$$\left. \begin{aligned} V_g^+ &= (V_2 + c_2 n_x^+)\boldsymbol{e}_x + c_2 n_y \boldsymbol{e}_y, \\ n_x^+ = \frac{(\hat{q}^2-1)^{1/2} - \hat{q}M_2}{\hat{q} - M_2(\hat{q}^2-1)^{1/2}}, &\quad n_y^+ = \frac{(1-M_2^2)^{1/2}}{\hat{q} - M_2(\hat{q}^2-1)^{1/2}}. \end{aligned} \right\} \tag{2.19}$$

Relations (2.19) can be derived from (2.10).

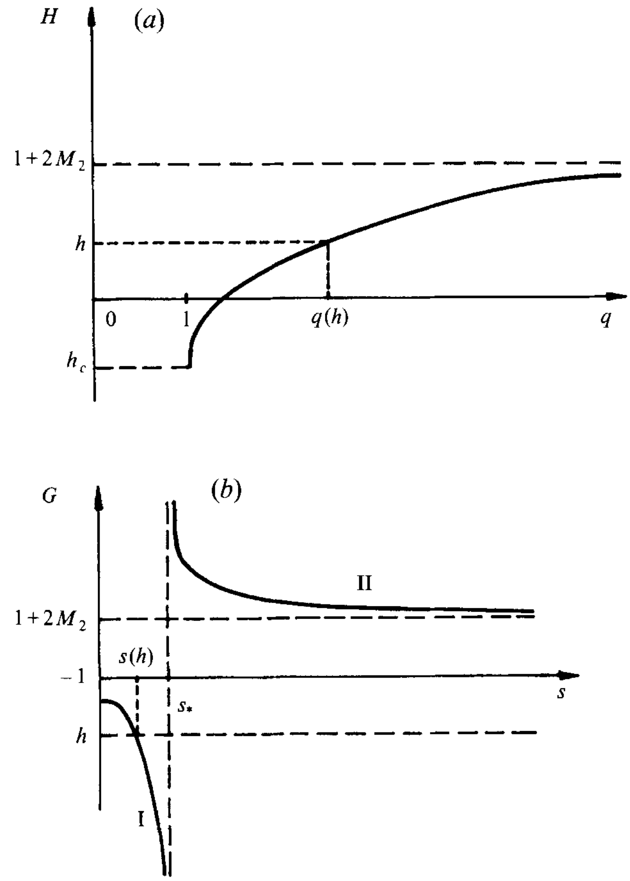

FIGURE 3. (*a*) Schematic view of the dependence $h = H(q)$. Intersection of the curve $H(q)$ with the straight line $h = \text{const}$ determines the value $q(h)$ corresponding to an infinite value of the reflection coefficient for a real value of ω. (*b*) Schematic view of the dependence $h = G(s)$. The value $s(h)$ corresponds to an infinite value of the reflection coefficient for a purely imaginary value of ω.

The limiting case $h = h_c$ refers to the spontaneous emission of a wave with the critical frequency $\omega = \omega_c$. Such a wave propagates parallel to the shock surface, as follows from (2.19), i.e. $V^+_{gx} = 0$ for $\hat{q} = 1$. The upper boundary of the interval (2.16), $h = 1 + 2M_2$, corresponds to the spontaneous emission of a strictly one-dimensional wave: in the limit $q \to \infty$ we obtain $n^+_y = 0$ and $V^+_g = (V_2 + c_2)\,\boldsymbol{e}_x$.

Note that the condition for resonant reflection should be the same as the condition for resonant transmission of acoustic and entropy-vortex waves propagating from the supersonic region 1. It is evident that the solution representing only one outgoing wave $\mathscr{A}^+$ is the solution of any of the four problems (*a–d*) formulated in the previous subsection.

The corrugation instability of a shock wave was defined by Dyakov as the existence of solutions of the type of (2.9), for which

$$C^- = 0, \quad \operatorname{Im}\omega > 0, \quad \operatorname{Im} k^+_x > 0, \quad \operatorname{Im} k^{(v)}_x > 0, \quad \operatorname{Im} k_y = 0. \tag{2.20}$$

Such solutions represent the acoustic and entropy-vortex perturbations that decay in space away from the shock and grow in time. Dyakov's criteria for corrugation instability have the form

$$h < -1 \quad \text{or} \quad h > 1 + 2M_2. \tag{2.21}$$

Therefore, there are two regions of the h-axis where the corrugation instability may occur. We will show below that like the spontaneous emission phenomenon, the corrugation instability may be interpreted as a result of infinitely strong reflection. The waves that undergo such an infinite reflection belong to a special class of travelling waves. Let us consider the acoustic waves $\mathscr{A}^{\pm}$, for which the perturbations $\delta p^{\pm}$ are described by (2.9) with a purely imaginary value of $\omega = \mathrm{i}\lambda$. Assuming $\lambda > 0$, we obtain from (2.10)

$$k_x^{\pm} = -\mathrm{i}\chi^{\pm}, \quad \chi^{\pm} = \frac{\lambda[M_2 \mp (1+s^{-2})^{1/2}]}{c_2(1-M_2^2)}, \quad s^{-2} = \frac{\omega_c^2}{\lambda^2}. \tag{2.22}$$

The quantities $k_x^{\pm}$ determined by (2.22) are purely imaginary, while $\chi^+ < 0$, $\chi^- > 0$. Therefore, the perturbations (2.9) take the form

$$\delta p^{\pm} = C^{\pm} \exp(\lambda t + \chi^{\pm} x + \mathrm{i}k_y y). \tag{2.23}$$

The solution δp^- represents a wave that grows both in time and in the positive x-direction. Actually this wave propagates with the velocity $u^- = -\lambda/\chi^- < 0$ and, therefore, moves in the negative x-direction. The solution δp^+ represents a wave moving in the positive x-direction with the velocity $u^+ = -\lambda/\chi^+ > 0$. Thus, both acoustic waves $\mathscr{A}^-, \mathscr{A}^+$ decay in the directions of their propagation. Let us suppose that in region 2 there is an external source of perturbations of sufficiently large distance from the shock. Let such a source create a pressure perturbation growing in time as $\exp \lambda t$. We can say that the wave δp^- defined in (2.23) represents the part of the exciting signal that can reach the shock. After reflection of the wave δp^-, the reflected wave δp^+ will propagate from the shock, decaying in space and growing in time.

In accordance with the definition of the corrugation instability, the latter provides for the existence of the outgoing wave $\mathscr{A}^+$ without an incident wave $\mathscr{A}^-$. Therefore, we can expect that under the conditions (2.21) the reflection coefficient $\mathscr{R}(\omega, k_y)$ for waves having the form (2.23) takes an infinitely large value at some point of the complex ω-plane belonging to the positive imaginary semiaxis ($\omega = \mathrm{i}\lambda$). To check this proposition we set $\omega = \mathrm{i}\lambda$ in the formula (2.13) for $\mathscr{R}$ and consider the conditions under which the denominator on the right-hand side of (2.13) equals zero. We obtain the following relationship:

$$h = G(s, M_2), \tag{2.24}$$

where $$G(s, M_2) \equiv H(-\mathrm{i}q, M_2) = \frac{(1-M_2^2)[s^2+2M_2 s(s^2+1)^{1/2}] + \eta M_2^2}{(1-M_2^2)s^2 - \eta M_2^2}.$$

For real values of $s = \lambda/\omega_c$, the function G is real. A typical form of the dependence $G(s)$ is shown in figure 3(*b*). One can see that G decreases monotonically from the value $G = -1$ to $G = -\infty$ when s varies within the interval $(0, s_*)$, and G decreases monotonically from the value $G = +\infty$ to $G = 1+2M_2$ when s varies within the interval (s_*, ∞). The value $s = s_* = [\eta/(1-M_2^2)]^{1/2} M_2$ represents a simple pole of the function $G(s)$. It is evident that there exists a unique value of s in the interval $(0, \infty)$ satisfying (2.24) if h belongs to one of two regions of corrugation instability (2.21).

For a given value of k_y, the growth rate λ in the first region of corrugation instability ($h < -1$) increases from the value $\lambda = 0$ at $h = -1$ to the value $\lambda_* = \omega_c s_*$ at $h = -\infty$. In the second region ($h > 1+2M_2$), λ undergoes the same variation (from $\lambda = 0$ at $h = 1+2M_2$ to $\lambda_* = \omega_c s_*$ at $h = \infty$).

Thus, we have found that for waves of the form (2.23) the reflection coefficient $\mathscr{R}$ becomes singular in the region of corrugation instability. A further consideration shows that there are no other growing waves except (2.23) (say, waves with $\mathrm{Im}\, \omega > 0$

and Re $\omega \neq 0$), for which $\mathscr{R}$ can be made infinitely large. In this connection we note that for all real values of ω satisfying (2.18), the behaviour of $\mathscr{R}$ is regular in both regions of corrugation instability and, furthermore, $0 < \mathscr{R} \leqslant 1$ in the first Dyakov region ($h < -1$). Therefore, it would be impossible to predict the corrugation instability in this region considering the reflection of periodical acoustic waves. It should be noted, however, that within the second Dyakov region ($h > 1+2M_2$) the value of $|\mathscr{R}|$ may become greater than 1 for periodical acoustic waves.

3. Shock adiabatics for ionizing shock waves

3.1. *Consequences of the conservation laws*

Consider the propagation of a plane shock wave with constant velocity in a cold non-ionized monatomic gas. When the shock Mach number M_1 is sufficiently large, a partially ionized plasma is created behind the shock. The concentration of electrons increases in the relaxation zone and reaches a constant value in the equilibrium region as is shown in figure 2. The kinetics of the ionization depends on the type of gas as well as on the initial state of the gas ahead of the shock. In a certain thin layer directly behind the shock the ionization starts with atom–atom collisions. When the ionization degree grows, electron–atom collisions dominate the ionization process and play a crucial role in further increasing the electron density. A discussion of the effects of the various elementary processes on the ionization relaxation behind strong shock waves can be found in the review by Biberman, Mnatsakanyan & Iakubov (1970).

In order to obtain an equation for the shock adiabatic we use the relationships that connect the values of the parameters of the gas in regions 1 and 2:

$$\rho_1 V_1 = \rho_2 V_2 = j, \tag{3.1}$$

$$p_1 + jV_1 = p_2 + jV_2, \tag{3.2}$$

$$\epsilon_1 + \frac{p_1}{\rho_1} + \frac{V_1^2}{2} = \epsilon_2 + \frac{p_2}{\rho_2} + \frac{V_2^2}{2} + Q. \tag{3.3}$$

Here ϵ is the internal energy of the gas per unit of mass, and $Q > 0$ is the loss of energy due to ionization per unit of mass. Relations (3.1)–(3.3) represent the conservation of mass, momentum and total energy of an ionizable gas passing through the shock and the relaxation zone. For a pure monatomic gas, the quantities ϵ_1, ϵ_2 and Q are written in the form

$$\epsilon_1 = \frac{p_1}{(\gamma-1)\rho_1}, \quad \epsilon_2 = \frac{p_2}{(\gamma-1)\rho_2}, \quad Q = \frac{NI\alpha}{\mu}, \tag{3.4a–c}$$

with $\gamma = C_p/C_v = 5/3$. Here I is the first ionization potential, N is the Avogadro number, μ is the atomic weight, and α is the degree of ionization defined as

$$\alpha = n_e/(n_i + n_a), \tag{3.5}$$

where n_e, n_i and n_a are the concentrations of electrons, positive ions, and neutral atoms, respectively. Under the equilibrium condition, the equality $n_e = n_i$ is satisfied.

Relations (3.4) are true for the simplest model of an ionizing shock that neglects the excitation of electron levels of atoms and takes into account only the first ionization of the atoms. Such an approach is both illustrative and convenient in analysing the shock adiabatic. The restrictions of this model are discussed in §6.

The expressions for ϵ_2 given by (3.4*b*) can be justified as follows. In the equilibrium state the mean energy per particle, regardless of its charge and mass, equals $\frac{3}{2}kT_2$, where

T_2 is the temperature and k is the Boltzmann constant. Therefore, the initial energy ϵ_2 per unit of mass is determined as

$$\epsilon_2 = \tfrac{3}{2}kT_2(n_e+n_i+n_{a2})/\rho_2.$$

The total pressure in the mixture of atoms, electrons and ions is determined by the formula

$$p_r = kT_2(n_e+n_i+n_{a2}).$$

Comparing the last two formulae, we get $\epsilon_2 = \tfrac{3}{2}p_2/\rho_2$.

To obtain an equation describing the ionizing shock adiabatic we express the degree of ionization in terms of the thermodynamic parameters using the Saha equation. The latter presents the condition of an ionization equilibrium and has the following form (Mitchner & Kruger 1973):

$$\frac{\alpha^2}{1-\alpha} = \frac{g}{n_i+n_a}\left[\frac{m_e kT_2}{2\pi\hbar^2}\right]^{3/2} \exp\left(-\frac{I}{kT_2}\right). \tag{3.6}$$

Here m_e is the mass of an electron, and $\hbar$ is Planck's constant. The factor g in the Saha equation is determined as $2\Sigma_i/\Sigma_a$, where Σ_i and Σ_a are the statistical sums of the ion and the atom, respectively. The quantity g usually is of order 1–10. Thus, for argon $g = 11$ can be assumed in the range of temperatures $T < 15000$ K (Mitchner & Kruger 1973).

After the transition to the dimensionless variables

$$\theta = T_2/T_1, \quad \eta = \rho_2/\rho_1 \tag{3.7}$$

the solution α of the quadratic equation (3.6) is presented in the form

$$\left.\begin{aligned} &\alpha(\eta,\theta) = -\tfrac{1}{2}\psi + (\tfrac{1}{4}\psi^2+\psi)^{1/2},\\ &\psi = b(p_1,T_1)\,\theta^{3/2}\eta^{-1}\exp(-\zeta/2\theta), \quad \zeta = 2I/(kT_1),\\ &b(p_1,T_1) = g(m_e/2\pi\hbar^2)^{3/2}(kT_1)^{5/2}/p_1. \end{aligned}\right\} \tag{3.8}$$

Using the conservation laws (3.1)–(3.3) together with the relations (3.4), (3.7), we obtain the equation of the shock adiabatic $\eta = \eta(P)$ in the implicit parametric form

$$\eta(\theta) = \frac{1+4P(\theta)}{4+P(\theta)-\zeta\alpha(\eta(\theta),\theta)}, \quad P(\theta) = [1+\alpha(\eta(\theta),\theta)]\,\theta\eta(\theta), \tag{3.9}$$

where the function $\alpha(\eta,\theta)$ and the parameter ζ are defined in (3.8). The contribution of the ionization degree α to the dimensionless pressure P reflects the contribution of the electron pressure p_e in the total pressure of a partially ionized plasma behind the shock.

3.2. *Non-monotonic behaviour of the ionizing shock adiabatic*

Relation (3.9) determines an implicit function $\eta(P)$. It is evident that $\eta(P)$ represents a single-valued function, since the right-hand side of (3.9) is a monotonically decreasing function of η for an arbitrary fixed value of P. For the case of a shock propagating in a cold ionizable gas, $\eta(P)$ has a maximum at a certain value $P = P_m$. In the range of values $P = O(1)$, $\eta(P)$ is growing, since it is close to the classic shock adiabatic for a regular gas, while in the range $P \gg \zeta$ we get the asymptotic formula

$$\eta \approx 4(1+\zeta/P), \tag{3.10}$$

which shows that η should decrease when P is increased. Therefore, $\eta(P)$ should have a maximum.

In obtaining the asymptotic formula (3.10) it was assumed that $\zeta \gg 4$. This inequality

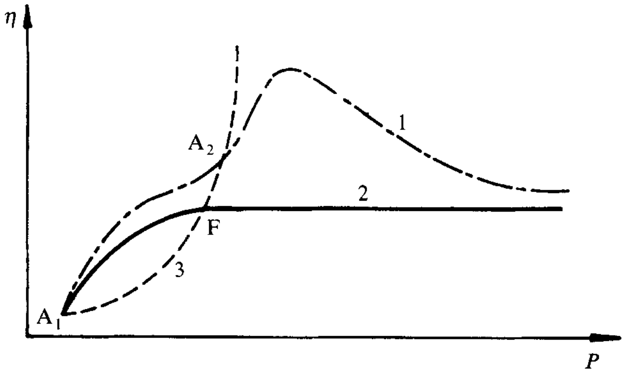

FIGURE 4. Mutual arrangement of the ionizing shock adiabatic (1), the gasdynamic shock adiabatic (2) and the Rayleigh line (3).

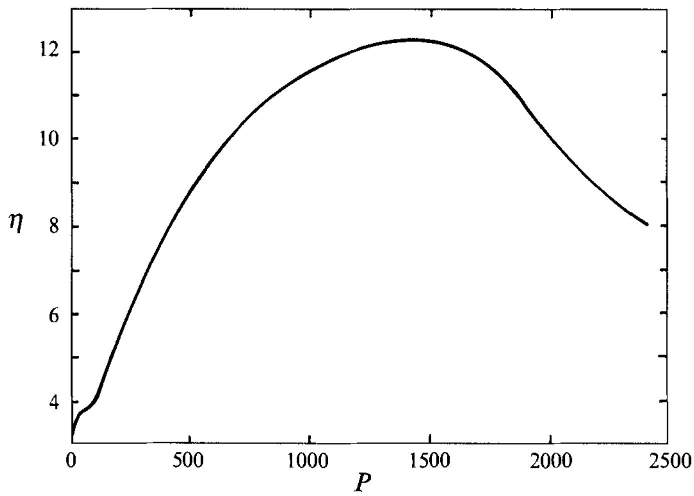

FIGURE 5. Ionizing shock adiabatic for argon with $T_1 = 300$ K, $p_1 = 5$ Torr.

is indeed satisfied. Thus, for $T_1 = 300$ K, the values of ζ are of order of 10^2 for alkali metal vapours and of order of 10^3 for inert gases. For example, in caesium and argon, the values of ζ are 301 and 1219, respectively.

In spite of the fact that the denominator on the right-hand side of (3.9) vanishes along a certain line in the (η, P)-plane, the dependence $\eta(P)$ determined by (3.9) is regular and bounded from above. If η were singular at some value of P, then there would be a contradiction because in accordance with (3.8), $\alpha \rightarrow 0$ when $\eta \rightarrow \infty$. Therefore, the denominator in (3.9) cannot vanish simultaneously with an infinite increase in the quantity η.

A typical form of an ionizing shock adiabatic is shown schematically in figure 4 (curve 1). The numerical solution of (3.9) for argon is presented in figure 5.

The occurrence of a decreasing section on a shock adiabatic in the presence of endothermic phase transitions, such as dissociation or ionization, can be explained as follows. For a sufficiently high intensity of the shock, there is a considerable relative drop of the temperature in the relaxation zone $(T_F - T_2)/T_F$ (here T_F is the frozen temperature behind the gasdynamic shock) associated with the losses of thermal energy by an endothermic reaction (see figure 2). This leads to an increase in the gas density, while ρ_2 exceeds the maximal value of density which is attainable in a perfect gas,

$\rho_2 = 4\rho_1$. On the other hand, for very large values of the shock intensity ($P \gg \zeta$), the frozen temperature T_F is so large, that the ratio T_2/T_F remains close to unity in spite of the loss of energy in the relaxation zone. It means that ρ_2 should tend to the value $4\rho_1$ as if the ionization were absent. Thus, we obtain the decreasing dependence $\eta(P)$ described by the asymptotic expression (3.10).

It is known that the degree of compression produced by ionizing shocks may considerably exceed the classic gasdynamic limit $\eta = 4$. Such higher compression degrees were indeed observed by Griffiths *et al.* (1976) and Glass & Liu (1978), and obtained in the numerical calculations of the non-equilibrium region behind the gasdynamic shock in argon (Glass & Li 1978; Kaniel *et al.* 1986; Liberman & Velikovich 1986). Analysing the equations that describe the structure of relaxation zone, Liberman & Velikovich noticed that the degree of compression from shocks can reach very high values, although they did not consider the global properties of the ionizing shock adiabatic. In our numerical calculations the value $\eta_{max} = 12.3$ was obtained for argon at $T_1 = 300$ K and $p_1 = 5$ Torr. The lowering of p_1 resulted in larger values of the compression degree.

We also note that the evolutionarity condition $M_2 < 1$ is satisfied along the shock adiabatic everywhere.

3.3. *The Rayleigh curve and the relaxation zone*

The process of transition from the initial gas state 1 to the final equilibrium state 2 may be depicted in the (P, η)-plane in the following way. For a given Mach number ahead of the shock M_1, the state of the gas immediately behind the gasdynamic shock (the frozen state) is given by the point F which represents the intersection of the usual shock adiabatic (curve 2 in figure 4) and the Rayleigh curve determined by the equation

$$\eta_R = \frac{\gamma M_1^2}{1+\gamma M_1^2 - P} \tag{3.11}$$

with $\gamma = \frac{5}{3}$. The hyperbolic curve (3.11) represents all states in the (P, η)-plane which are attainable in a stationary flow from an initial state 1 when the mass flow rate j and the total momentum $p + jV$ are conserved. These two conservation laws are satisfied across the gasdynamic shock as well as across the relaxation zone. Therefore, the final state 2 of a partially ionized gas is determined by the point A_2, at which the Rayleigh curve 3 intersects the ionizing shock adiabatic as shown in figure 4. The consideration of the ionizing shock transition in the (P, η)-plane is analogous to the representation of a detonation wave given in Landau & Lifshitz (1987), who considered the detonation shock adiabatic in the plane (P, V) where $V = 1/\eta$ is the normalized specific volume. In the plane (P, V) equation (3.11) describes a straight line termed the Rayleigh line. Like the detonation shock adiabatic, the ionizing shock adiabatic, generally, does not pass through the initial point A_1 if the shock propagates in a preheated gas with non-zero initial ionization degree α_1. In a cold gas with $T_1 \leqslant 300$ K, it can be assumed that $\alpha_1 = 0$. Then both curves 1 and 2 in figure 4 pass through the point A_1. This condition is satisfied for the shock adiabatic given by (3.9), though in figure 5 the initial section of shock adiabatic going out from the point $P = 1$, $\eta = 1$ is not shown.

The significance of the Rayleigh curve in analysing the shock transition to the plasma state is in the fact that the motion from F to A_2 along this curve shows the direction in (P, η)-plane for which the changes of the variables η and P in the relaxation zone are consistent with conservation laws for mass and momentum. Generally, the Rayleigh curve may intersect the ionizing shock adiabatic in several points if the latter has several maxima. An oscillating character of $\eta(P)$ occurs in the range of very high values of P where the ionization of single- and multi-charged ions occurs (Kuznetsov 1989). Within the framework of the model adopted, only one maximum on the shock adiabatic exists and only one point on the A_2-type (see figure 4) belongs both to the shock adiabatic and to the Rayleigh curve.

The existence of a unique intersection point A_2 does not always mean that the state A_2 really can be reached. For the realization of this state, it is necessary that the direction of variation of the parameters η and P along the section of Rayleigh curve FA_2 is confirmed by the stationary solution that describes the relaxation zone. To justify the transition FA_2 one has to check that the final gas temperature T_2 at the point A_2 is lower than the frozen temperature T_F at the point F, as was predicted in calculations by Glass & Liu (1978) and Kaniel *et al.* (1986). For the ionizing shock adiabatic described by (3.9), the requirement $T_F > T_2$ is satisfied, so that the transition $A_1 FA_2$ is consistent with real structure of the relaxation zone.

4. Acoustic waves in a partially ionized gas

4.1. *Perturbations in the equilibrium region behind the shock*

In analysing the conditions for spontaneous acoustic emission from an ionizing shock the simple relationship between the pressure and the density perturbation

$$\delta p = \frac{5kT}{3m_a}\delta\rho, \tag{4.1}$$

where m_a is the atomic mass, can no longer be used for acoustic waves in a partially ionized plasma. When an acoustic wave propagates in such a plasma, the density and temperature perturbations of the heavy particles (atoms and positive ions) give rise to perturbations in the ionization degree $\delta\alpha$ and in the electron temperature δT_e. Although all species of the plasma have the same unperturbed temperature in the equilibrium state in region 2, the perturbations of the electrons' temperature δT_e and of the heavy particles' temperature δT are different. This difference may be considerable when the period of the acoustic oscillations is less than the characteristic time of the energy exchange between the heavy particles and the electrons.

The full system of linearized equations that describe small perturbations in a partially ionized plasma is very cumbersome. Since our purpose is in finding the range of parameters for which the resonant reflection ($R = \infty$) may occur, we select below only the principal mechanisms that are responsible for this phenomenon.

The steady state behind the ionizing shock represents a uniform flow with $V_e = V$, $n_e = n_i$ and $T_e = T$. We consider now perturbations whose frequency, ω, satisfies the following conditions:

$$\omega \ll 2\nu_{ion}, \quad \omega \gg \nu_\epsilon, \quad \omega\tau_M \ll 1, \quad \omega \ll \nu_e, \quad \omega \ll V_2/d, \tag{4.2a–e}$$

where ν_{ion} is the characteristic frequency of ionization by electron impact, ν_ϵ is the inverse of the characteristic time for the energy transfer from heavy particles to electrons, τ_M is the Maxwellian relaxation time for the electric charge density, ν_e is the effective frequency of momentum transfer from heavy particles to electrons, and d is the characteristic lengthscale of the density variations within the relaxation zone. Conditions (4.2b) and (4.2d) can be simultaneously satisfied since

$$\nu_\epsilon = 2\frac{m_e}{m_a}\alpha\nu_e \ll \nu_e. \tag{4.3}$$

Condition (4.2*a*) means that the period of the acoustic oscillations is much larger than the time of transition to the ionization equilibrium determined by the actual electron temperature T_e. As a result, the Saha equation (3.6) can be employed in order to calculate the perturbed electron concentration.

As a result of condition (4.2*b*) the energy exchange between the electrons and the heavy particles due to elastic collisions can be neglected in the energy equation of the latter. However, further assumptions are needed in order to neglect that term in the energy equation for the electrons, as will be discussed below.

Conditions (4.2*a*) and (4.2*b*) are of crucial importance to the occurrence of spontaneous emission of acoustic waves from an ionizing shock. Thus, for waves with ω which is much larger than the upper limit set by (4.2*a*), the kinetics of the electrons can be ignored and the waves propagate in effectively a perfect gas. As is well known, under such circumstances the shock is stable. On the other hand, if ω is much smaller than the limit set by condition (4.2*b*), $\delta T_e = \delta T$ and the acoustic waves propagate in an effectively single gas in which the electrons and the heavy particles are in local thermodynamic equilibrium. As will be shown later, this limit also results in an ionizing shock that is stable for spontaneous emission of sound.

Condition (4.2*c*) results in the quasi-neutrality of the plasma, namely $|\delta n_e - \delta n_i| \ll |\delta n_e|$, where δn_e and δn_i are the perturbations of the concentrations of the electrons and the ions, respectively. The Maxwellian relaxation time is given by

$$\tau_M = \frac{\epsilon_0}{\sigma} = \frac{\epsilon_0 m_e \nu_e}{e^2 n_e}, \tag{4.4}$$

where ϵ_0 is the absolute dielectric permittivity and σ is the electric conductivity of the plasma in equilibrium.

Condition (4.2*d*) means that during the period of the acoustic oscillations an electron undergoes many collisions with the heavy particles, so that the inertial terms in the electron momentum equation may be neglected. Usually this condition is satisfied in a collisional plasma behind the shock.

Discussion of condition (4.2*e*) is deferred to the next subsection where it will be used. A numerical example of the various limits in (4.2) is given in §6.

Under conditions (4.2) the system of equations describing the perturbations of the electrons' variables is given, after linearization, by conservation of the total number of free and bounded electrons

$$\frac{\partial \delta n_e}{\partial t} + \nabla \cdot (n_e \delta V_e + \delta n_e V_e) = \delta \dot{n}_e, \tag{4.5}$$

the momentum equation

$$-\nabla \delta p_e + m_e \nu_e n_e (\delta V - \delta V_e) + e n_e \delta \boldsymbol{E} = 0, \tag{4.6}$$

the energy equation

$$\frac{\partial}{\partial t}(\tfrac{3}{2} n_e k \delta T_e + \tfrac{3}{2} \delta n_e k T_e) + \nabla \cdot [\tfrac{3}{2} k T_e (n_e \delta V_e + \delta n_e V_e) + \tfrac{3}{2} k \delta T_e n_e V_e]$$
$$= -p_e \nabla \cdot \delta V_e - I \delta \dot{n}_e - \frac{2m_e}{m_a} \nu_e n_e \tfrac{3}{2} k (\delta T_e - \delta T). \tag{4.7}$$

In these equations $\delta \dot{n}_e$ is the perturbation of the source term in the rate equation for the electron concentration, $e < 0$ is the electron charge and $\delta \boldsymbol{E}$ is the perturbation of the electric field. Although the perturbation of the electric charge density created by an acoustic wave does not vanish identically (as well as the electric field perturbation $\delta \boldsymbol{E}$), we can replace δn_i by δn_e in the linearized equations, when necessary, as a result of assumption (4.2c).

Before continuing, we notice that since ω is assumed to be much less than the effective frequency of momentum transfer between ions and atoms we may set $\delta V_i = \delta V_a$. Furthermore, as a result of the latter equality and of condition (4.2c) the conservation law for the electric charge is given by

$$\nabla \cdot \delta j = \nabla \cdot en_e(\delta V_e - \delta V_i) = 0.$$

As a result we obtain

$$\nabla \cdot \delta V_e = \nabla \cdot \delta V_a. \tag{4.8}$$

We assume that in region 2 the ionization takes place due to the electron–atom collisions, while the recombination is due to the triple electron–electron–ion collisions, so that the direct and inverse processes are represented as

$$\mathrm{e} + \mathrm{a} \leftrightarrow 2\mathrm{e} + \mathrm{i}.$$

The nonlinear source term $\dot{n}_e$ within the framework of such an assumption has the following form (Mitchner & Kruger 1973):

$$\dot{n}_e = \nu_{ion} n_e - \kappa_{rec} n_e^2 n_i, \tag{4.9}$$

where ν_{ion} is the effective ionization frequency and κ_{rec} is the triple recombination coefficient. In the equilibrium state the right-hand side of (4.9) is zero. The ratio ν_{ion}/ρ as well as κ_{rec} are functions of T_e. Therefore, the perturbation $\delta \dot{n}_e$ can be written as

$$\delta \dot{n}_e = -2\nu_{ion}\,\delta n_e + n_e \frac{\partial \nu_{ion}}{\partial \rho}\delta\rho + n_e\left(\frac{\partial \nu_{ion}}{\partial T_e} - n_e^2 \frac{\partial \kappa_{rec}}{\partial T_e}\right)\delta T_e. \tag{4.10}$$

The right-hand side of (4.10) is of the order of

$$2\nu_{ion}\,\delta n_e \approx 2\nu_{ion}\frac{n_e}{T_e}\left(\frac{\partial \ln n_e}{\partial \ln T_e}\right)_{eq}\delta T_e \approx \nu_{ion}\frac{n_e}{T_e}\left(\frac{I}{kT_e}\right)\delta T_e. \tag{4.11}$$

Here $(\partial n_e/\partial T_e)_{eq}$ is the derivative of the electron density, $n_e(T_e, \rho)$, as determined by the Saha equation with respect to $T_e = T$. Estimate (4.11) is valid under condition (4.2a).

Assuming that condition (4.2a) is valid, we can neglect the perturbation of the energy loss due to elastic collisions (the third term on the right-hand side of (4.7)) as compared with the energy loss by ionization (the second term on the right-hand side of (4.7)) when the following inequality is satisfied:

$$\frac{3m_e}{m_a}\frac{\nu_e}{\nu_{ion}}\left(\frac{kT_e}{I}\right)^2 \ll 1. \tag{4.12}$$

The equations that describe the perturbations in the heavy-particles gas are

$$\left(\frac{\partial}{\partial t} + V \cdot \nabla\right)\delta\rho + \rho \nabla \cdot \delta V = 0, \tag{4.13}$$

$$\left(\frac{\partial}{\partial t} + V \cdot \nabla\right)\delta V = -\frac{\nabla(\delta p_H + \delta p_e)}{\rho}, \tag{4.14}$$

$$\left(\frac{\partial}{\partial t} + V \cdot \nabla\right)\left[\frac{\delta p_H}{p_H} - \frac{5}{3}\frac{\delta\rho}{\rho}\right] = 0. \tag{4.15}$$

Here $\delta p_H = \delta p_a + \delta p_i$ is the pressure perturbation and $\delta V = \delta V_i = \delta V_a$ is the velocity perturbation in the heavy-particles gas. The perturbation of electron pressure δp_e appears in the momentum equation for heavy particles (4.14) as a result of eliminating the electric field perturbation from the momentum equation for the electrons (4.6) and the momentum equation for ions. The latter has the form

$$m_i n_i\left(\frac{\partial}{\partial t}+V_i\cdot\nabla\right)\delta V_i = -\nabla\delta p_i - en_i\,\delta E - m_e \nu_{ei} n_e(\delta V_i - \delta V_e)$$
$$= -\nabla(\delta p_i + \delta p_e) + m_e \nu_{ea} n_e(\delta V_a - \delta V_e). \quad (4.16)$$

Here ν_{ei} and ν_{ea} are the effective frequencies of the electron–ion and electron–atom collisions, respectively, so that $\nu_e = \nu_{ei} + \nu_{ea}$. The perturbed momentum equation for neutral atoms is

$$m_a n_a\left(\frac{\partial}{\partial t}+V_a\cdot\nabla\right)\delta V_a = -\nabla\delta p_a - m_e n_e \nu_{ea}(\delta V_a - \delta V_e). \quad (4.17)$$

Equation (4.14) represents the sum of two equations, (4.16) and (4.17), while in obtaining this sum the equalities

$$\nu_e = \nu_{ei} + \nu_{ea}, \quad m_i = m_a, \quad n_i = n_e, \quad V_i = V_a = V$$

were used.

Finally, using (4.7), (4.8), (4.12) and (4.13) the energy equation for electrons can be rewritten as

$$\left(\frac{\partial}{\partial t}+V\cdot\nabla\right)\left[\tfrac{3}{2}\delta p_e + I\delta n_e - (\tfrac{5}{2}p_e + In_e)\frac{\delta\rho}{\rho}\right] = 0. \quad (4.18)$$

We are now ready to modify equation (4.1) such that the electrons' kinetics is taken into account. To this end we start with (4.18) which yields

$$\frac{\delta p_e}{p_e} = \left(\frac{5}{3}+\frac{2I}{3kT}\right)\frac{\delta\rho}{\rho} - \frac{2I\delta n_e}{3kTn_e}. \quad (4.19)$$

As follows from the definitions of the degree of ionization $\alpha = n_e/(n_e+n_i)$ and the gas density $\rho = m_a(n_a+n_i)$,

$$\frac{\delta n_e}{n_e} = \frac{\delta(\alpha\rho)}{\alpha\rho} = \frac{\delta\rho}{\rho}+\frac{\delta\alpha}{\alpha}. \quad (4.20)$$

In addition, the following relations will be used:

$$\left.\begin{aligned} &\frac{\delta p_e}{p_e} = \frac{\delta(\rho\alpha T_e)}{\rho\alpha T_e} = \frac{\delta\rho}{\rho}+\frac{\delta\alpha}{\alpha}+\frac{\delta T_e}{T}, \\ &\frac{\delta\alpha}{\alpha} = \bar{\alpha}_T\frac{\delta T_e}{T}+\bar{\alpha}_\rho\frac{\delta\rho}{\rho}, \quad \bar{\alpha}_T = \left(\frac{\partial \ln\alpha}{\partial \ln T}\right)_\rho, \quad \bar{\alpha}_\rho = \left(\frac{\partial \ln\alpha}{\partial \ln\rho}\right)_T. \end{aligned}\right\} \quad (4.21)$$

It should be noted that in obtaining equation (4.21) it has been assumed that α depends on the electron temperature. This assumption stems from the fact that for sufficiently high degrees of ionization in the region behind the shock ($\alpha > 10^{-3}$–10^{-2}) the dominant ionization mechanism is the electron–atom collision. Since condition (4.2*a*) is satisfied, the logarithmic derivatives in (4.21) should be calculated for the equilibrium dependence $\alpha(T,\rho)$ determined by the Saha equation (3.6). Elimination of the quantities $\delta n_e/n_e$, $\delta\alpha/\alpha$, and $\delta T_e/T$ from the system of relations (4.19)–(4.21) results in the following relationship:

$$\frac{\delta p_e}{p_e} = A\frac{\delta\rho}{\rho}, \quad A = \frac{(5+z)(\bar{\alpha}_T+1)-z\bar{\alpha}_\rho}{3(\bar{\alpha}_T+1)+z\bar{\alpha}_T}, \quad z = \frac{2I}{kT} = \frac{\zeta}{\theta}. \tag{4.22}$$

As a result of (4.15) and condition (4.2*b*) the adiabatic equation with $\gamma = \frac{5}{3}$,

$$\frac{\delta p_H}{p_H} = \frac{5\delta\rho}{3\rho}, \tag{4.23}$$

may be used for the perturbations of density and pressure created by an acoustic wave in the gas of atoms and ions. However, the analogous conclusion is not true for the equation that connects $\delta\rho$ with the total pressure perturbation δp and determines both sound velocity and the effective adiabatic index γ_{eff}. Taking into account the relations $p_e = \alpha p/(1+\alpha)$, $p_H = p/(1+\alpha)$ and $\delta p = \delta p_e + \delta p_H$, we obtain from (4.22) and (4.23) the following relationship:

$$\frac{\delta p}{p} = \gamma_{eff}\frac{\delta\rho}{\rho}, \quad \gamma_{eff} = \frac{5}{3}\left(\frac{1+\frac{3}{5}\alpha A}{1+\alpha}\right) \tag{4.24}$$

where A is defined in (4.22).

4.2. *The reflection coefficient for acoustic waves from ionizing shocks*

We now return to the concept of ionizing shock developed in §2 and consider the reflection of acoustic waves from such a shock. An acoustic wave, which propagates in region 2 behind the relaxation zone, is described by the closed system of equations (4.13), (4.14) and (4.24). This system represents the equations of acoustics of a moving uniform medium, in which the sound velocity is determined as

$$c_2 = (\gamma_{eff}p_2/\rho_2)^{1/2}. \tag{4.25}$$

Therefore, the Mach number M_2 entering in the formulae of §2 will be defined by the expression

$$M_2^2 = \frac{\rho_2 V_2^2}{\gamma_{eff}(\rho_2, T_2)p_2}. \tag{4.26}$$

The definition of the parameter h given in (2.14) also should be modified for the following reason. In order to derive the reflection coefficient $\mathscr{R}$, the jump conditions between the variables on both sides of the shock should be used. For steady state, the jump conditions are given by (3.8) and (3.9) and can be written symbolically as

$$\rho_2 = \mathscr{F}(p_2; \rho_1, p_1). \tag{4.27}$$

For classic gasdynamic shocks, for which ionization is practically zero, taking into account that the perturbations on the supersonic side of the shock are zero yields

$$\rho_2 + \delta\rho_2 = \mathscr{F}(p_2+\delta p_2; \rho_1, p_1), \tag{4.28}$$

which after linearization results in

$$\delta\rho_2 = \left(\frac{\partial\rho_2}{\partial p_2}\right)_{\rho_1, p_1}\delta p_2. \tag{4.29}$$

The reason why the steady-state relationships can be used also for the perturbed state, (4.28), lies in the fact that within the diminishingly small shock width, the spatial gradients are arbitrary large, hence the time derivatives can be neglected in the conservation equations.

For strong shocks, however, the spatial gradients within the relaxation zone (which in our model is equivalent to the shock width) are finite. Hence, the steady-state relations across the shock (i.e. across the relaxation zone) can be used for the perturbed state only when the frequency is small enough, according to condition (4.2*e*).

Generalizing (4.29) to the case of ionizing shocks, it is now given by

$$\delta\rho_2 = \left[\left(\frac{\partial\rho_2}{\partial p_2}\right)_{\rho_1,\,p_1,\,\alpha} + \left(\frac{\partial\rho_2}{\partial\alpha}\right)_{\rho_1,\,p_1,\,p_2}\frac{\delta\alpha}{\delta p_2}\right]\delta p_2. \tag{4.30}$$

For calculating the quantity $\delta\alpha/\delta p_2$ we make of use (4.19)–(4.21), which result in the expression

$$\frac{\delta\alpha}{\delta p_2} = \frac{B\alpha}{p_2}, \quad B = \frac{3(1+\alpha)\,[\bar{\alpha}_\rho + \bar{\alpha}_T(A-1)]}{5(1+\bar{\alpha}_T)\,(1+\frac{3}{5}\alpha A)}. \tag{4.31}$$

Here, $\bar{\alpha}_\rho, \bar{\alpha}_T$ and A are defined in (4.21) and (4.22), respectively, and calculated at $\rho = \rho_2, T = T_2$.

Thus, a new definition of the parameter h is obtained as follows:

$$h = -V_2^2\left[\left(\frac{\partial\rho_2}{\partial p_2}\right)_{\rho_1,\,p_1,\,\alpha} + \frac{\alpha B}{p_2}\left(\frac{\partial\rho_2}{\partial\alpha}\right)_{\rho_1,\,p_1,\,p_2}\right]. \tag{4.32}$$

As before, the general expression for the reflection coefficient has the form (2.13) with the distinction that the quantities c_2, M_2 and h now are determined by the relations (4.25), (4.26) and (4.32), respectively. Therefore, the critical value of the parameter $h = h_c$ is determined by (2.16), in which the Mach number M_2 is calculated in accordance with (4.26).

5. Spontaneous emission of sound from ionizing shocks: numerical results

Numerical calculations of the parameters h and h_c determined by (4.32) and (2.16) with allowance for (4.26) were carried out for different monatomic gases at different initial pressures. An equation of the form of (3.9) was used for the ionizing shock adiabatic. Examples of such calculations for an ionizing shock in argon are shown in figures 6 and 7. As is seen from these figures, there is a threshold with respect to the shock Mach number M_1, beyond which the condition for spontaneous emission $h > h_c$ is satisfied. This theoretical result is in good agreement with the experiments of Glass & Liu (1978) with argon, and Glass *et al.* (1977) with krypton, in which the development of instabilities accompanied by oscillations of the gas density was observed. Glass *et al.* also studied the influence of small hydrogen impurities on the structure of the relaxation zone and the perturbations behind the shock. We do not comment here on the effects of impurities since their inclusion in the kinetic model is outside the framework of the present paper. The focus of our attention is on the experimental results for pure gases.

The occurrence of instabilities in pure argon and krypton has been observed for sufficiently high values of M_1. In the experiments of Glass *et al.* the concentration of electrons and the gas density throughout the flow were determined by using a Mach–Zehnder interferometer with a ruby laser light source. The shock wave velocity

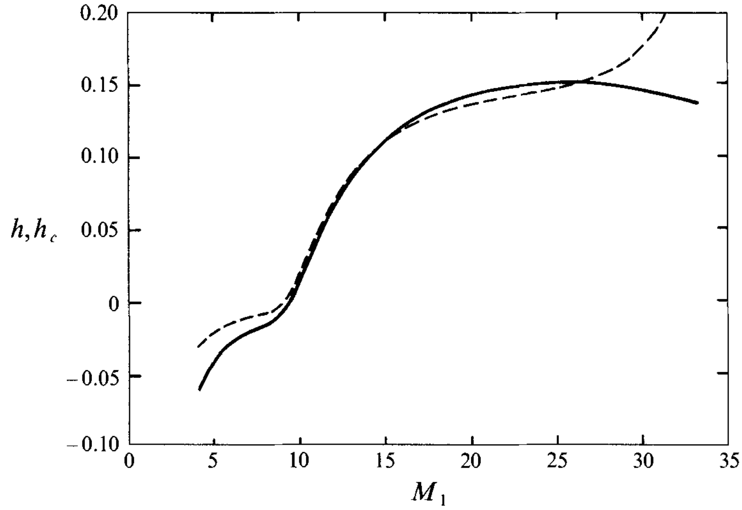

FIGURE 6. Dependencies $h(M_1)$ (——) and $h_c(M_1)$ (----) for an ionizing shock in argon with $T_1 = 300$ K, $p_1 = 5$ Torr.

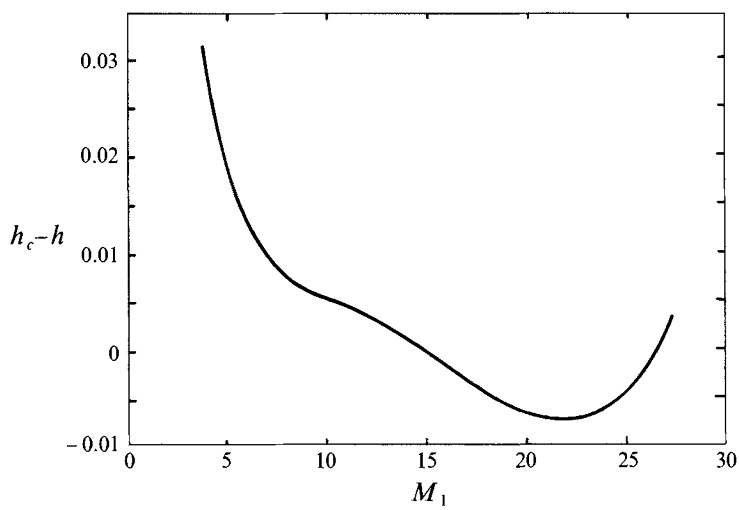

FIGURE 7. The difference $h_c - h$ as a function of the Mach number M_1 for an ionizing shock in argon at $T_1 = 300$ K, $p_1 = 5$ Torr.

and the pressure measurements through the shock wave were performed by means of pressure transducers located at several positions along the hypervelocity shock tube. An oscillating structure of the flow downstream of the shock wave front was seen from interferograms for supercritical values of M_1. Thus, in argon at $p_1 = 5$ Torr, $T_1 = 300$ K the instabilities appeared for $M_1 \geqslant 14.7$. As can be seen from figure 7, the threshold of spontaneous emission obtained in our calculations is $M_1 = M_c = 15$. This comparison indicates that the present theoretical model is in good quantitative agreement with experiment.

According to the numerical calculations, spontaneous emission occurs in a finite (but sufficiently wide) interval of the shock Mach number values: $M_c < M_1 < M^*$. This interval expands if the initial density of the gas ahead of the shock is decreased. Thus, for argon at $T_1 = 300$ K, the boundaries of the region of spontaneous emission on the

M_1-axis are given by $M_c = 15$, $M^* = 26.6$ and $M_c = 13.8$, $M^* = 27.3$ for $p_1 = 5$ Torr and 1 Torr, respectively. The existence of an upper boundary for spontaneous emission, $M_1 = M^*$, was not observed experimentally: the maximal value of the shock Mach number presented in Glass & Liu (1978) is 17.7, which is within the theoretical range of spontaneous emission.

It also should be noted that in all our calculations the values of parameter h were inside the interval $(-1, 1)$, so that Dyakov's conditions of corrugation instability was never reached.

6. Discussion

In this paper the frequency-dependent coefficient of reflection $\mathscr{R}$ of two-dimensional acoustic waves from a plane shock was considered as an appropriate tool for analysing the stability of shock waves. Expression (2.13) determining $\mathscr{R}$ as an explicit function of the ratio ω/k_y and the parameters M_2 and h gives the simplest form of representation of the reflection coefficient. This form can be employed in the analysis of the various boundary-value problems connected with the reflection of sound waves from shocks.

Expression (2.13) is rigorous in the framework of a single-fluid description of acoustic waves in a spatially uniform flow behind the shock. For non-uniform flows, the expression obtained is valid within the WKB-approximation, i.e. for short waves satisfying the condition $|k_x| \, l \gg 1$, where l is the characteristic length of the variation of the flow parameters. The same formula may be applied to the reflection of a short acoustic wave from a shock with a finite radius of curvature. Such an approach was employed in the consideration of the reflection of sound waves propagating in a plane spiral flow behind the cylindric shock wave (Rutkevich & Mond 1992).

We have demonstrated in §2 the usefulness of the representation (2.13) in deriving the conditions for spontaneous emission and for corrugation instability. The occurrence of the latter was interpreted as the condition of resonant reflection ($\mathscr{R} = \infty$) for a special class of acoustic waves decaying in space.

The model of a non-monotonic ionizing shock adiabatic developed in §3 describes only the principal mechanism for the energy losses in the relaxation zone, namely the thermal ionization of the neutral atoms. The dependent $\eta(P)$ would be calculated with better accuracy if the loss of energy by radiation and excitation of the electron levels of the atoms were included in the model. At the same time, we found that the values of the degree of ionization calculated from the system of equations (3.8) and (3.9) are in a good agreement with experimental data. Thus, for argon at $T_1 = 300$ K, $p_1 =$ 5 Torr and $M_1 = 16.1$ we obtained $\alpha = 0.15$. Under these condition, the same value of α was measured by Glass & Liu (1978) immediately behind the relaxation zone. Further slow decrease of α downstream, which was observed in that experiment, was neglected in our model.

More serious restrictions of the present model exist in the range of high temperatures $kT_2 \geqslant 0.1I$ corresponding to the descending section of the shock adiabatic, since in this range of temperatures the energy losses by the ionization of multi-charged ions influence the qualitative behaviour of $\eta(P)$. Such a multiple ionization may result in the appearance of a sequence of maxima of the function $\eta(P)$. Nevertheless, the increasing section of the shock adiabatic will be described satisfactorily by the single-ionization model due to the condition

$$(I_2/I) \exp\left[(I-I_2)/kT_2\right] \ll 1,$$

where I_2 is the second ionization potential.

Analysis of acoustic perturbations superimposed on the equilibrium flow behind the relaxation zone can be applied to various problems associated with the acoustics of partially ionized plasmas created behind ionizing shocks that propagate in monatomic gases. This analysis has shown that there is a range of acoustic frequencies ω in which the relative perturbations of the electron gas density and temperature are much less than the analogous perturbations of the heavy-particle gas parameters, i.e.

$$\frac{\delta n_e}{n_e} = O\left(\frac{\delta \rho}{\rho}\right), \quad \frac{\delta T_e}{T_e} \ll \frac{\delta T}{T}. \qquad (6.1\,a, b)$$

According to (4.18), which expresses the balance of the electron energy in the perturbed state, such a balance is held if $\delta\rho/\rho$ is of the order of $\delta n_e/n_e$ since $z = 2I/kT$ is a large parameter. Then, from (4.20) and (4.22), in the limit $z \gg 1$ we obtain

$$\frac{\delta \alpha}{\alpha} \ll \frac{\delta \rho}{\rho}. \qquad (6.2)$$

The equilibrium degree of ionization $\alpha(T_e, \rho)$ is very sensitive to small variations of the electron temperature due to the inequality

$$\bar{\alpha}_T \approx \tfrac{1}{4} z \gg 1. \qquad (6.3)$$

Taking into account (6.3) and the relation $\delta\rho/\rho \approx 3\delta T/2T$, which is fulfilled for the acoustic wave, we obtain (6.1b) from the relation (6.2).

Conditions (6.1) and (6.3) are of crucial significance for the occurrence of spontaneous acoustic emission. To see this point we start with the asymptotic expressions for perturbations in the limit $z \gg 1$ that may be obtained from (4.19)–(4.21) and from the relations $\bar{\alpha}_T \approx \frac{1}{4}z$, $\bar{\alpha}_\rho \approx -\frac{1}{2}$ (for $\alpha \ll 1$):

$$\frac{\delta T_e}{T_e} \approx \frac{2\delta\rho}{z\rho}, \quad \frac{\delta n_e}{n_e} \approx \frac{\delta\rho}{\rho}. \qquad (6.4)$$

In the same limit $z \gg 1$, the quantities A, γ_{eff} and B entering (4.24)–(4.26), (4.31) have the form

$$A \approx 1 + \frac{8}{z}, \quad \gamma_{eff} \approx \frac{5 + 3\alpha(1 + 8/z)}{3(1+\alpha)}, \quad B \approx \frac{18(1+\alpha)}{z(5+3\alpha)}. \qquad (6.5\,a\text{–}c)$$

In order to understand why spontaneous emission may occur on the increasing section of the shock adiabatic we consider the variations of the parameters h and h_c when the shock Mach number M_1 increases.

For sufficiently small intensity of the shock both h and h_c are negative like in the case of a perfect gas. At $M_1 \approx 10$, the parameter h_c vanishes and becomes positive for larger values of M_1 (see figure 6). This behaviour of h_c is explained as follows. Using (2.16), (3.2) and (4.26), we can present h_c in the form

$$h_c = \frac{[(\gamma_{eff} - 1)\eta - (\gamma_{eff} + 1)]P + \eta + 1}{(\eta - 1)[(\gamma_{eff} + 1)P - 1]}. \qquad (6.6)$$

In the region $M_1 > 5$, the inequality $P \gg \eta$ holds, so that the sign of h_c is determined by the sign of $(\gamma_{eff} - 1)\eta - (\gamma_{eff} + 1)$. As follows from (6.5$b$), the value γ_{eff} is close to $\frac{5}{3}$ when $\alpha \ll 1$. Therefore, h_c should be positive when η slightly exceeds the value $\eta = 4$ corresponding to maximal degree of compression in a perfect gas with $\gamma = \frac{5}{3}$. As is seen from figure 6, it occurs at $M_1 = 10$.

Now consider the expression (4.32) for the parameter h. As follows from the shock adiabatic equation (3.9), the partial derivatives entering the right-hand side of (4.32) are determined by

$$\left(\frac{\partial \rho_2}{\partial p_2}\right)_{\rho_1,\, p_1,\, \alpha} = \frac{\rho_1(15-4\zeta\alpha)}{p_1(P+4-\zeta\alpha)^2}, \quad \left(\frac{\partial \rho_2}{\partial \alpha}\right)_{\rho_1,\, p_1,\, p_2} = \frac{\rho_1\,\zeta(4P+1)}{(P+4-\zeta\alpha)^2}. \tag{6.7}$$

According to (6.5c), the quantity B is small when $\alpha \ll 1$. Therefore, the term containing the first of the two derivatives in (6.7) dominates in the right-hand side of (4.32). This first term changes sign from negative to positive when the ionization degree α exceeds the value $15/4\zeta \ll 1$. Thus, h still becomes positive on the rising section of the shock adiabatic approximately in the same region of the M_1 values where the parameter h_c becomes positive. The competition between these two parameters may lead to the situation where h becomes larger than h_c and the region of spontaneous emission occurs on the M_1-axis as shown in figures 6 and 7.

In order to show that the assumptions of §4 can be satisfied we consider the following example. Let an ionizing shock propagate in argon at $M_1 = 16.5$, $p_1 =$ 5 Torr, and $T_1 = 300$ K. For such a shock the calculated values of the parameters entering into the criterion of spontaneous emission and in inequalities (4.2) and (4.12) are

$$z = 2I/kT_1 \approx 28, \quad M_2 \approx 0.45, \quad h \approx 0.12 > h_c,$$
$$2\nu_{ion} \approx 10^6\ \mathrm{s}^{-1}, \quad \nu_\epsilon \approx 8\times 10^4\ \mathrm{s}^{-1}, \quad \tau_M \approx 2\times 10^{-11}\ \mathrm{s}^{-1}, \quad \nu_e \approx 2\times 10^{10}\ \mathrm{s}^{-1},$$
$$V_2/d \approx 10^6\ \mathrm{s}^{-1}.$$

Inequality (4.12) is satisfied since its right-hand side is of order of 10^{-3}. To fulfil the set of conditions (4.2) we can assume that $\omega \approx (2-4)\times 10^5\ \mathrm{s}^{-1}$.

The density perturbations behind the relaxation zone are transferred away from the ionizing shock by two spontaneously emitted waves: the downstream acoustic wave and the entropy wave. The ratio of the amplitudes of density perturbations for these two waves is determined by

$$\delta\rho^{(s)}/\delta\rho^+ = -1-hM_2^{-2}. \tag{6.8}$$

In the example given above, $1+hM_2^{-2} \approx 1.6$; therefore, the density perturbation in the entropy wave $\delta\rho^{(s)}$ prevails over $\delta\rho^+$. In addition, the spatial period $L^{(s)}$ of oscillations $\delta\rho^{(s)}$ is considerably smaller than the wavelength L^+ of the acoustic oscillations. Thus, for $\omega = 3\times 10^5\ \mathrm{s}^{-1}$ the wavelength of the entropy-vortex perturbation is $L^{(s)} \approx 2$ cm, while $L^+ \approx 8$ cm. These estimates allow us to suggest that the gas density oscillations observed by Glass & Liu (1978) represent spontaneously emitted entropy waves rather than acoustic ones.

In the limit of low frequencies ω satisfying the condition

$$\omega \ll \frac{3m_e \nu_e \alpha}{m_a \bar{\alpha}_T} \tag{6.9}$$

the difference between the electron temperature and heavy-particle temperature perturbations cannot be supported any longer over the period of oscillations. Under condition (6.9), one can assume $\delta T_e = \delta T$, and the parameter h takes the classic form given in (2.14), while the derivative $(\partial\rho_2/\partial p_2)_{\rho_1,\, p_1}$ is calculated in accordance with the equation of the ionizing shock adiabatic (3.9). As a result, the coefficient B in (4.32) will not be that given by (4.31). This new coefficient $B = B_{eq}$ is determined as

$$B_{eq} = (\mathrm{d}\ln\alpha/\mathrm{d}\ln p_2)_{\rho_1,\, p_1}. \tag{6.10}$$

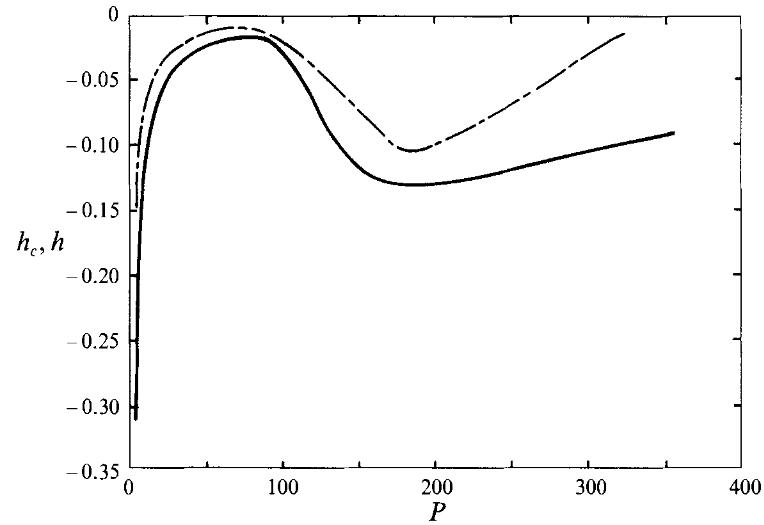

FIGURE 8. Quasi-stationary dependencies $h(P)$ (——) and $h_c(P)$ (–·–) calculated under the assumption $\delta T_e = \delta T$ for an ionizing shock in argon with $T_1 = 300$ K, $p_1 = 55$ Torr.

The derivative in (6.10) is taken along the equilibrium shock adiabatic. The quantity γ_{eff} also differs from the previous one given by (4.24) and now is determined as

$$\gamma_{eff} = \frac{5[1+\alpha+\frac{1}{5}z(\bar{\alpha}_T-\bar{\alpha}_p-1)\alpha]}{3(1+\alpha+\frac{1}{3}z\bar{\alpha}_T\alpha)}. \tag{6.11}$$

Unlike the coefficient B in the non-equilibrium model, the quantity B_{eq} is not small. The parameter h cannot change sign on the increasing section of the shock adiabatic because it is just proportional to the total derivative $d\rho_2/dp_2$ along $\eta = \eta(P)$ given by (3.9).

The typical behaviour of $h(P)$ and $h_c(P)$ in the equilibrium case is presented in figure 8, which shows that within the equilibrium model with $\delta T_e = \delta T$ the difference $h - h_c$ is negative. Therefore, the ionizing shock should be stable to low-frequency perturbations. The range in which the stability can be guaranteed depends on the initial conditions and on the Mach number M_1. Thus, for $M_1 = 16.5$, $p_1 = 5$ Torr, and $T_1 = 300$ K, the resonance reflection of acoustic waves cannot take place in the range $\omega < 1000\ s^{-1}$. For a given value of ω condition (6.9) may be reached by raising the collision frequency ν_e by means of raising the initial pressure.

In the limit of high-frequency waves, for which $\nu_{ion} \ll \omega \ll \nu_e$, the kinetics of ionization does not influence the acoustic waves, and the latter are propagating in a plasma as in a regular gas. An incident acoustic wave passes the relaxation zone and reflects from the gasdynamic shock with reflection coefficient $|\mathscr{R}| < 1$. Therefore, in the high-frequency limit the spontaneous emission should disappear again. Thus, the frequencies of waves that can be spontaneously emitted by strong ionizing shocks are bounded both from below and above.

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REFERENCES

ANDERSON, J. D. 1989 *Hypersonic and High Temperature Gas Dynamics*. McGraw-Hill.

BIBERMAN, L. M., MNATSAKANYAN, A. KH. & IAKUBOV, I. T. 1970 Ionization relaxation behind strong shock waves in gases. *Sov. Phys. Usp.* **102** (3), 431–462.

BRILLOUIN, J. 1953 Reflexion et refraction d'ondes acoustique par une onde de choc. *Acustica* **5** (3), 149–163.

DYAKOV, S. P. 1954 On the stability of shock waves. *Zh. Eksp. Teor. Fiz.* **27** (9), 288–295.

FOWLES, G. R. 1981 Stimulated and spontaneous emission of acoustic waves from shock fronts. *Phys. Fluids* **24**, 220–227.

GLASS, I. I. & LIU, W. S. 1978 Effects of hydrogen impurities on shock structure and stability in ionizing monatomic gases. Part 1. Argon. *J. Fluid Mech.* **84**, 55–77.

GLASS, I. I., LIU, W. S. & TANG, F. C. 1977 Effects of hydrogen impurities on shock structure and stability in ionizing monatomic gases: 2. Krypton. *Can. J. Phys.* **55**, 1269–1279.

GRIFFITHS, R. W., SANDEMAN, R. J. & HORNUNG, H. G. 1976 The stability of shock waves in ionizing and dissociating gases. *J. Phys.* D: *Appl. Phys.* **9**, 1681–1691.

KANIEL, A., IGRA, O., BEN-DOR, G. & MOND, M. 1986 Ionization behind strong shocks in argon. *Phys. Fluids* **29**, 3618–3625.

KONTOROVICH, V. M. 1957 On the stability of shock waves. *Zh. Ekspr. Teor. Fiz.* **33**, 1525–1526.

KONTOROVICH, V. M. 1959 Reflection and refraction of sound by shock waves. *Akust. Zh.* **5** (3), 314–323.

KUZNETSOV, N. M. 1989 Stability of shock waves. *Sov. Phys. Usp.* **32** (11), 993–1012.

LANDAU, L. D. & LIFSHITZ, E. M. 1987 *Fluid Mechanics*, 2nd Edn. Pergamon.

LIBERMAN, M. A. & VELIKOVICH, A. L. 1986 *Physics of Shock Waves in Gases and Plasmas*. Springer.

MITCHNER, M. & KRUGER, C. H. 1973 *Partially Ionized Gases*. John Wiley & Sons.

RUTKEVICH, I. M. & MOND, M. 1992 Amplification of fast magnetosonic waves and the cutoff spectrum. *J. Plasma Phys.* **48**, 345–357.